## HEAT AND MASS TRANSFER IN DISPERSIVE AND POROUS MEDIA

## ANALYSIS OF MASS-TRANSFER PHENOMENA IN THE DENDRITIC NET OF SOLIDIFYING STEEL INGOTS. 1. FORMULATION OF THE PROBLEM. ANALYSIS OF MELT FLOW IN THE DENDRITIC NET

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Based on the solution of the problem of mass transfer in a dendritic net, an analysis of the motion of a melt in the interdendritic space and of mass-transfer phenomena in the moving melt has been performed with the aim of determining the growth in the concentration of a soluble impurity in the axial zone of solidifying continuous castings.

The manifestation of chemical inhomogeneity in steel ingots (off-center liquation and axial chemical inhomogeneity) is largely dependent on the movement of the melt in the interdendritic space of the two-phase zone of the ingots. According to Yu. D. Smirnov [1], it is precisely the melt flow along the crystallization front of large forginggrade ingots that determines the occurrence and development of off-center liquation (A-segregation).

For continuous steel castings, the occurrence of axial chemical inhomogeneity is related to melt flow in the two-phase zone in the direction of shrinkage zones on the axes of the ingots solidifying last [2-5]. Melt motion in the two-phase zone of solidifying ingots, which is formed by the interlacing of the dendritic crystallites with the melt, has been proved experimentally [6-8] and is confirmed by the results of computer modeling [9-11].

In [5], the occurrence of axial chemical inhomogeneity of continuous castings is assumed to be determined by two processes: separation diffusion at the phase boundary in the zone of solidification of the melt and the inflow of the melt enriched with a soluble impurity and necessary for making up for the shrinkage in the solidification zone to the shrinkage zone. Accordingly, growth in the concentration of the soluble impurity on the ingot axis is assumed to consist of two parts: a "diffusional" part due to the process of separation diffusion and a "translational" one determined by the transfer of the impurity with the melt flow to the shrinkage zone.

The present work seeks to analyze the "translational" component of the concentration growth in the axial zone of solidifying continuous castings. This problem is logically subdivided into two parts: an analysis of the melt motion in the interdendritic space and an analysis of mass-transfer phenomena in the moving melt and of the corresponding growth in the concentration of the soluble mixture. In the first part of the work, we study the velocity field of a melt in the dendritic net for a certain simplified scheme of mutual position of dendritic branches. Results of the analysis of diffusion processes in a melt are presented in the second part.

In evaluating the average velocity of movement of a melt in the two-phase zone of a solidifying ingot, we use the notion of the "permeability" of a dendritic net; the permeability values have been established empirically for a number of nonferrous metals and steel [9, 12]. The results of experiments on determination of the permeability of model alloys (Al–Sn and Pb–Sn) have been presented by A. I. Veinik [12, Chapter 3]; also, he has generalized the experimental data of other researchers for 35L steel and AL4 alloy. According to A. I. Veinik, the average values of the permeability coefficient for the above alloys vary within comparatively narrow limits ( $K = 10^{-11}-10^{-12}$  m<sup>2</sup>). The

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TABLE 1. Values of the Velocity of Flow U (cm/sec) of an Iron Melt in a Channel of Length 25 cm for Different Values of Permeability and Pressures at the Boundaries

$K_1, m^2$	$P_{x=0} = 0$ and $P_{x=0.25} = -10^5$ Pa	$P_{x=0} = 2 \cdot 10^5$ Pa and $P_{x=0.25} = -10^5$ Pa	$P_{x=0} = 2 \cdot 10^5$ Pa and $P_{x=0.25} = -2 \cdot 10^5$ Pa
10 <sup>-9</sup>	6.5	19	27
$10^{-10}$	0.65	1.9	2.7
$10^{-11}$	0.065	0.19	0.27
10 <sup>-12</sup>	0.0065	0.019	0.027



Fig. 1. Diagram of separation of the computational element ABCD in the system of two neighboring dendritic crystallites.

experiments of Japanese researchers [9] made it possible to establish the dependence of the permeability coefficient of carbon steel on the fraction of the liquid phase  $\varphi$  in the two-phase system dendritic crystallites-melt in the form of the following relations:

$$K = 10^{-7} + (\varphi - 0.85) \cdot 10^{-4}, \quad \varphi > 0.85; \quad K = 10^{-9}, \quad 0.85 > \varphi > 0.7; \quad K = 3.25 \cdot 10^{-10} \varphi^3, \quad \varphi < 0.7.$$
(1)

In [5], experimental data on the permeability coefficient of the ingot's two-phase zone have been used for evaluation of the velocity of movement of a melt based on numerical integration of the equation of motion

$$\rho \frac{\partial \mathbf{U}}{\partial t} - \mu \nabla^2 \mathbf{U} + \left(\frac{\mu}{K}\right) \mathbf{U} + \nabla p = \mathbf{F}.$$
<sup>(2)</sup>

In particular, in analyzing the stationary distribution of the melt-flow velocity in a plane channel of length 0.25 m, we have obtained the flow-velocity values presented in Table 1, when the pressures  $P_{x=0}$  and  $P_{x=0.25}$  were prescribed at the channel boundaries. It follows from these data that the velocities of flow of the melt vary within 0.1–1 mm/sec for the permeabilities of the two-phase zone  $K = 10^{-11} - 10^{-12} \text{ m}^2$ , which is consistent with the data of computer modeling [9, 11]. In the calculations, we used the values of the mass density of the melt  $\rho = 7000 \text{ kg/m}^3$  and the coefficient of dynamic viscosity  $\mu = 0.0062 \text{ Pa-sec}$ . The average flow velocities indicated were used in subsequent calculations (for a more detailed description of the velocity profile in the interdendritic space) as the input initial values of the velocity vector.

In a mathematical formulation of the problem of movement of a melt in a dendritic net, we will proceed from the following assumptions:

(a) an analysis is confined to the motion of the melt in the zone of columnar crystallites characterized by the existence of a system of mutually parallel axes of first order and axes of second order perpendicular to them;

(b) the melt flow obeys standard conditions for incompressible viscous fluid flow with the use of the Navier–Stokes equation resulting from a more general equation (2) when  $K \rightarrow \infty$  is prescribed;

(c) Eq. (2) is solved for the computational element ABCD located between two neighboring axes of first order (Fig. 1); within the computational element, there are five branches of second order that are an obstacle to the melt motion along the x axis;



Fig. 2. Pattern of melt flows between dendrite branches and layout of control cross sections: 1-4) two-phase zone; 5) boundary of the two-phase zone; 6) liquid phase. *y* and *x*, m.



Fig. 3. Streamlines of a melt between dendrite branches, when  $U_0 = 0.1$  mm/sec,  $\rho = 7000$  kg/m<sup>3</sup>, and  $\mu = 0.0062$  Pa·sec are prescribed. y and x, m.

(d) on the portions of contact of the melt with the dendrite branches, we use the "sticking condition" well known in hydrodynamics;

(e) the initial (inlet) velocity of flow of the melt  $U_0$ , whose values vary within 0.1–1 m/sec in accordance with the considerations expressed earlier, is assumed to be prescribed at the entrance into the computational interval ABCD (along the AB side).

The aim of the calculations is to determine the velocity profile of melt flow on the portions in contact with secondary dendrite branches, which is necessary for further investigation of the washing-out of the impurity and its involvement in the melt moving in the interdendritic space. The calculations with the finite-element method are carried out with variation of the dimension of branches of first and second order but for their invariant layout.

An analysis of the dynamics of time variation in the components of the velocity vector of the flow has shown that a stationary velocity field is established in the computational element ABCD within a few seconds (about 10 sec).

The computation results enable us to draw two conclusions:

(1) the laminar regime of flow is characteristic of the melt motion in the interdendritic space for the inlet velocities indicated;

(2) as the length of the branches of second order increases, the channels in which the melt flows converge, and we observe the acceleration of metal flows in accordance with hydraulic laws.

Figure 2 shows the pattern of melt flow for the case where we have the convergence of the vertices of branches of second order growing toward each other (the dendrite parameters are taken to be  $L_1 = L_2 = 1$  mm). Also, this figure gives the control cross sections A<sub>1</sub>B<sub>1</sub>, CD, and C<sub>1</sub>D<sub>1</sub> in which the profile of the longitudinal component of the flow-velocity vector U(x, y), when the flow regime becomes stationary, is fixed.



Fig. 4. Distribution of the flow velocity U along the  $A_1B_1$  line (see the diagram in Fig. 2). U, m/sec; x, m.



Fig. 5. Distribution of the flow velocity U along the CD (1) and C<sub>1</sub>D<sub>1</sub> lines according to the layout in Fig. 2. U, m/sec; y, m.

The flow pattern presented in Fig. 2 points to the fact that the flow of the melt flowing down from vertex D of the upper dendrite branch acquires, on the portion CD, a flow velocity exceeding the initial value  $U_0$ . As a result a melt flow with a higher-than-average velocity flows into the neighboring (lower) secondary branch of the dendrite, which produces certain changes in the process of mass exchange in the surface layer of the dendrite branch.

Figure 3 gives the streamlines of the melt in the interdendritic space which preserve a certain similarity with change (increase) in the length of dendrite branches of second order. Figure 4 shows the flow-velocity distribution U(x) along the line A<sub>1</sub>B<sub>1</sub>, and Fig. 5 shows the flow-velocity distribution U(y) in the control cross sections CD and C<sub>1</sub>D<sub>1</sub>. The plots in Fig. 5 confirm the fact of acceleration of the flow on the portions of convergence of the channels in the interdendritic space. It is seen that the velocity of flow of the melt grows from zero at the point of "sticking" to the surface of a dendrite branch up to the value  $U \cong 0.26$  mm/sec.

Increase in the velocity of the flow washing the surface of the dendrite branch contributes to the intensification of mass-exchange processes on this surface. In particular, the increase in the average velocity of the flow incident on the dendrite branch from 0.1 to 0.23 mm/sec leads to a growth of 40–50% in the mass-transfer coefficient of the dendrite surface. The discovery of this factor justifies the use of the complicated hydrodynamic model in analyzing mass-transfer phenomena in the interdendritic space of solidifying steel ingots.

## NOTATION

**F**, external-force vector; *K*, permeability coefficient of metals and alloys, m<sup>2</sup>;  $L_1$  and  $L_2$ , dendrite parameters, mm; *p*, local pressure, Pa;  $P_{x=0}$  and  $P_{x=0.25}$ , pressure at the boundaries of a plane channel, Pa; *t*, time, sec; **U**, flow-velocity vector; *U*, velocity of flow of the melt, cm/sec;  $U_0$ , initial (inlet) velocity of the melt, mm/sec; *x*, length of a plane channel, m; *y*, ordinate axis;  $\varphi$ , fraction of the liquid phase in the two-phase system dendritic crystalites-melt;  $\mu$ , coefficient of dynamic viscosity, Pa·sec;  $\rho$ , density, kg/m<sup>3</sup>;  $\nabla$ , Hamiltonian operator.

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